GR-Signature

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Abstract

In Computer Vision applications, objects can be discriminated based on their shapes. This paper presents a method to discriminate objects based on their shapes. The approach in this paper exploits Radon Transform and Gradient to simplify shape descriptors or signatures from an image. The simplified signatures are called GR-signatures. GR-signatures are used together with the proposed metric to estimate percentage of rectangularity of a given object. The percentage of rectangularity helps us to discriminate objects with low effort. The experiment has shown that GR-signature gives accurate results to measure rectangularity in comparison to approach proposed by Rosin [5]. The approach has low complexity and provides less sensitiveness to protrusions and noise.

Keywords: GR-signature, Shape descriptors, Radon Transform

Contents

1. Introduction

Object recognition is difficult task in computer vision. It is the task after the image segmentation task in image processing. Forms or shapes are analysed in this phase. Shapes analysis is applied in many areas such as in medicine (to detect anatomies), in computer-aided design, and in satellite image processing (to detect roads). We can distinguish objects based on their appearance, such as texture, color, and shape [2]. Many approaches of object recognition based on shapes measurement (circularity,ellipticity, and rectangularity) have been proposed, however the rectangularity approach has rare interest. In [2] and [5] presents shapes measurement based on rectangularity. They further introduce the aims of developing shape descriptors in the following:

- To develop a shape descriptor to make it invariant to certain transformations or variations
- To reduce noise sensitiveness
- To define a set of standard shapes (rectangles)
- To provide a global information for shapes to easy to distinguish

In this paper, we distinguish objects based on their shapes. A rectangle is decided as a set of standard shapes. First, we have to extract shapes from their images and simplify them to make them portable (easy to handle, store, and compare). To do so, we use Radon Transform to extract shapes and Gradient to make a signature called GR-signature [2]. The signature of an object represents its global information, which similarly to histogram of a digital image [6]. Then, we use GR-signatures and the rectangularity measurement metric to estimate percentage of rectangularity (R_{GR}). Rectangularity assumes the values from the interval zero to one ($\mathrm{R}_{GR} \in (0;1]$), with 1 represents a perfectly rectangular region [6]. Finally, we use these values to discriminate objects.

The remainder of the paper is structured as follows. Section 2 provides background information. It describes the basic terminology for object recognition such as how can we represent an image in computed tomography (CT). It further presents the techniques used to get GR-signature and describes how to determine rectangularity. Section 3 presents the evaluation of the GR-signature approach. It presents the results from two approaches (GR-signatures[2] and Rosin[5]). Finally, section 4 concludes the paper.

2. Methods

In this section, we describe the basic terminology for object recognition such as how can we represent an image in computed tomography (CT), how can we use Gradient get GR-signature, and how can we measure the rectangularity of a given object. We can use can use Radon Transform (RT) to recognize shapes, Gradient to locate the peaks (gradient profile) and find the modes of density. Finally, we can estimate the shape's rectangularity using GR-Signature to distinguish objects.

2.1. Radon Transform (RT)

The two-dimensional Radon Transform of an image defined in (x,y) coordinate space is a set of projections of the image taken by integrating along the set of lines defined by $x(cos\theta) + y(sin\theta) = \rho$, where $\theta \in [0; \pi)$ and $\rho \in (-\infty; \infty)$ [1]. The Radon Transform is given by:

$$\mathcal{R}(\rho, \theta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) \cdot \delta(\rho - x\cos\theta - y\sin\theta)\, dx\, dy \qquad (1)$$

In [4] describes the Radon Transform equation (1) in the following. The figure 1 illustrates the X-Ray beams. In computed tomography (CT), the lines are sent as X-Ray (beams) from the source through an object ($f(x, y)$) to the detector in parallel (families of parallel lines). Each beam has the same angle (θ). Then, we increase the angle until $\theta < \pi$ for each beam. We can use normal vector to describe the lines in the plane because they all have the same angles. A single line through the origin of the x and y axes (see p0 in the figure 1) is determined by its normal vector (also θ of its normal vector). The unit normal vector is given by $\underline{n} = (cos\theta, sin\theta)\, ;\, \theta \in [0; \pi)$. Further, a line that not through the origin (see p1 in the figure 1) can be determined by its distance from the origin(ρ). The lines can be distinguished by their signed distances(ρ). That is, if $\rho > 0$ the lines get from the origin in the direction of the normal vector, but $\rho < 0$ means the lines get from the origin to the direction opposite of the normal vector. Therefore, the range of the distance is defined by $\rho \in (-\infty; \infty)$. The families of parallel lines means θ is fixed and ρ is varied. Then, the line specified by the given pair (ρ, θ) can be represented in the cartesian equation in vector form as:

$$\underline{X} \cdot \underline{n} = \rho \implies x(cos\theta) + y(sin\theta) = \rho.$$

The line integral can be evaluated via the delta (δ) function (called line impulse). The definition of the delta function is given by: (see [8])

$$\delta(x) = \begin{cases} \infty & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases}$$

Consider, a delta function (δ) along the line described as:

$$\rho - x\cos\theta + y\sin\theta = 0.$$

That is

$$\delta(\rho - x\cos\theta + y\sin\theta).$$

The delta function is concentrated on the line; if its value is evaluated to zero (0) means it off the line, otherwise (∞) it is on the line, and can further integral. Therefore, we get the equation as in (1). For digital images, we can use an algorithm called "the approximate discrete Radon transform (ADRT)" to compute 4N-4 projections through an N x N image in $O(N^2 lgN)$ time. More details on this algorithm can be found in [1].

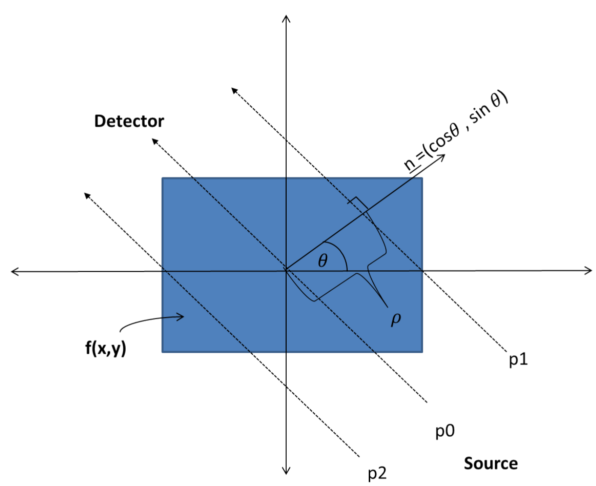


Figure 1: The figure represents parallel beams from a source to a detector through an object f(x,y) at the specific angle θ

2.2. Gradient

Radon transform contains many peaks, we use gradient to locate those peaks [2]. We compute the gradient (fixed θ) with respect to ρ to indicate the peaks (high and low) of Radon Transform $\mathcal{R}\,(\rho\ ,\ \theta)$. The general equation of gradient is given by

$$\vec{\nabla} f = \begin{pmatrix} \frac{\partial f(x,y)}{\partial x} \\ \\ \frac{\partial f(x,y)}{\partial y} \end{pmatrix} \tag{2}$$

The low and high peaks of Radon Transform (we fix θ and let ρ vary) can be estimated by gradient using this equation

$$\frac{\partial \mathcal{R}'(\theta, \rho)}{\partial \rho} \tag{3}$$

Please note that $\mathcal{R}'(\theta, \rho)$ represents the transposed matrix of Radon Transform $\mathcal{R}(\rho, \theta)$ described in the next section 2.3.

2.3. GR-Signature

A new GR-Signature is generated to be the global information of a given binary shape. Binary images are use to avoid the brightness caused by RT reflection that can be applied only with unique color. We perform these steps to get GR-signature:

1. Use a fast algorithm (O($N^2 lgN$)) [See 1] to transform Radon into Matrix $\mathcal{R} : N\rho * N_\theta$ cells
2. Transpose the Matrix R to change its dimension (e.g. $\mathcal{R}' : N_\theta * N\rho$)
3. Apply the equation (3) for each column
4. Extract the extrema each column (Maxima in positive side and Minima in negative side), we will get one dimension GR-Signature (see the figure 3d)
5. Select the peaks to estimate rectangularity (next section)

2.4. Rectangularity Measure Metric(R_{GR})

The R_{GR} is used to estimate the rectangularity of the given objects. The approach presented here are similar to the approach in the book of [6]. It is described in the following. If we know θ, we can calculate the length and the width of a rectangle by defining

$$\alpha(x, y) = xcos\theta + ysin\theta, \;\; \beta(x, y) = -xsin\theta + ycos\theta \tag{4}$$

Then, we search for the minimum and maximum of α and β over all boundary points(x,y). The bounding rectangle are defined by the values of α_{min}, α_{max}, β_{min}, and β_{max}. The length of the bound rectangle is determined by $\alpha_{max} - \alpha_{min}$ and its width is determined by $\beta_{max} - \beta_{min}$.

In this paper, we can measure rectangularity using R_{GR} metric in the following:

1. Angle Measurement (An)
2. Amplitude Measurement (Am)
3. Rectangularity Measure (R_{GR}) $= \frac{An+Am}{2}$

The equation (5) shows how the angle can be measured. Here, only four extrema are chosen (see section 2.3) regardless how many corner the shape has.

$$Angle\,Measurement = \frac{|90 - (\theta_{low} + \theta_{high})|}{90} \tag{5}$$

θ_{low} represents the difference between the two low peaks, whereas θ_{high} represents the difference between the two high peaks (See the figure 2). These peaks are estimated from GR-signature described in the section 2.3. The equation (6) shows how can the amplitude measurement be calculated. It has the range from [0; 1]. A_{max} represents the greatest amplitude, whereas A_{min} represents the smallest amplitude (See the figure 2). We need four extrema of the peaks to evaluate the amplitude. A_i is one amplitude of the four peaks $A_i(i \in [1..4])$ (ascend order).

Once, the four amplitudes are determined, we can apply the equation (7) to measure the amplitude of the rectangle. Finally, the shape rectangularity can be measured by using the equation (8).

$$A_i = \frac{A_i - A_{min}}{A_{max} - A_{min}} \tag{6}$$

$$Amplitude\,Measurement = 1 - \frac{|A_2 - A_1|}{|A_4 - A_3|} \tag{7}$$

$$R_{GR} = \frac{Angle\,Measurement + AmplitudeMeasurement}{2} \tag{8}$$

3. Evaluation

In this section, we examine the effectiveness of GR-signature and compare its results to the results from Ronsin [5].

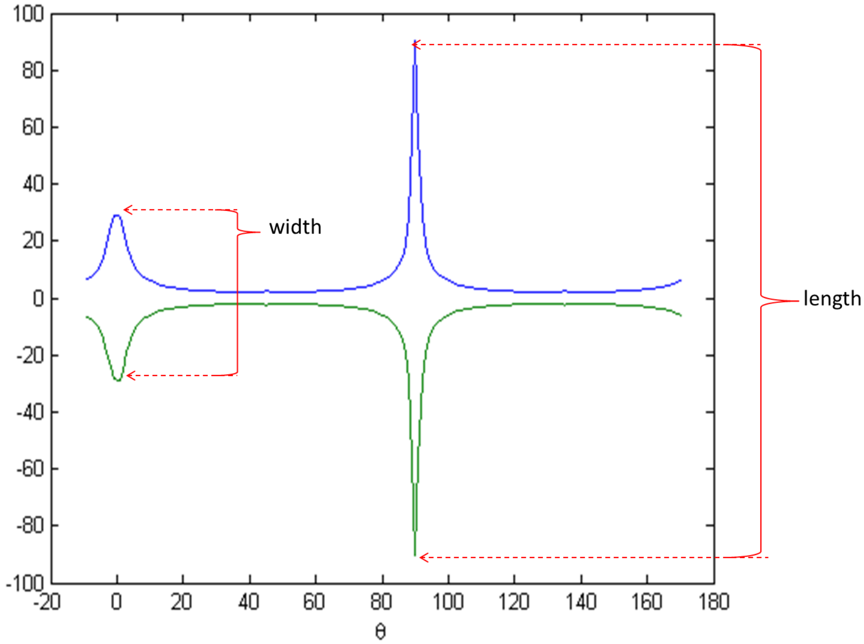
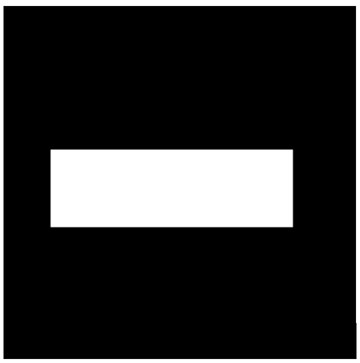
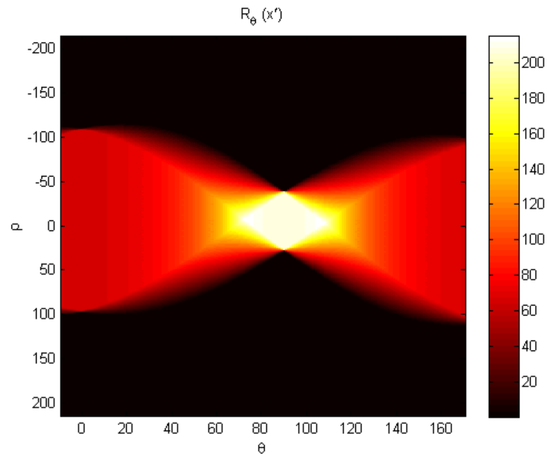


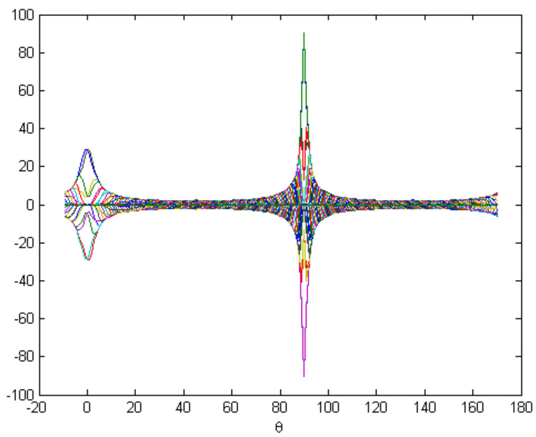
Figure 2: The figure represents the angle, the width, and the length of an ideal rectangle. The difference between two low/high peaks is 90 degree. Its height and its width are determined by the sum of two low/high peaks. Source: [2]



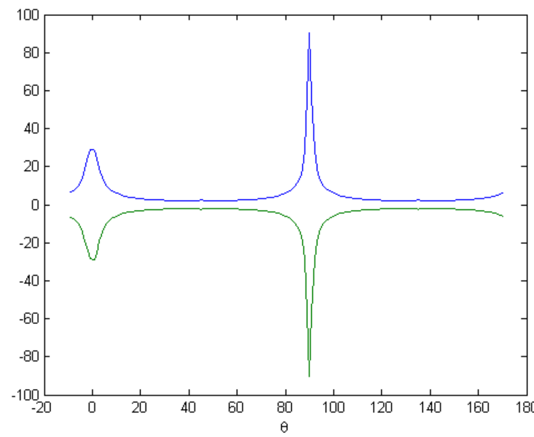
(a) A rectangle shape



(b) The signal of Radon Transform after projection ([0;180])



(c) Gradient result calculation



(d) GR-signature

Figure 3: GR-signature and its results. Source: [2]

3.1. Experimental Setup

To evaluate GR-signature approach, we have used synthetics shapes from an image database (the sources are undefined). First, we have applied the GR-signature approach to a rectangle shape. Then, we have observed the R_{GR} of it. In the experiment, we have observed the R_{GR} of the rectangle shape with several properties include: full shape, empty shape, translation, rotation, scaling, Gaussian noise, protrusions and indentations, and boundary noise. The results are shown in the figure 4. Second, we have applied GR-Signature to shapes from an image database. Then, we have compared the results to the results presented by Rosin [5]. Rosin has developed three rectangularity measures. Further, he has evaluated on both synthetic and real data. In [2] claims that some results of their work have obtained better results than the results from Rosin. The results are shown in the figure 5.

3.2. Results and Discussion

The significant founds are in the following :

- Full or empty shapes are unaffected to rectangularity measurement,we have got the same R_{GR} results by applying the GR-signature to both properties
- The descriptor (GR-Signature) is invariant under geometrical transformations (translation, rotation and scaling), we have got R_{GR} over 0.98
- The R_{GR} measurement is robust to the noise (by applying Gaussian noise , R_{GR} obtained over 0.97)

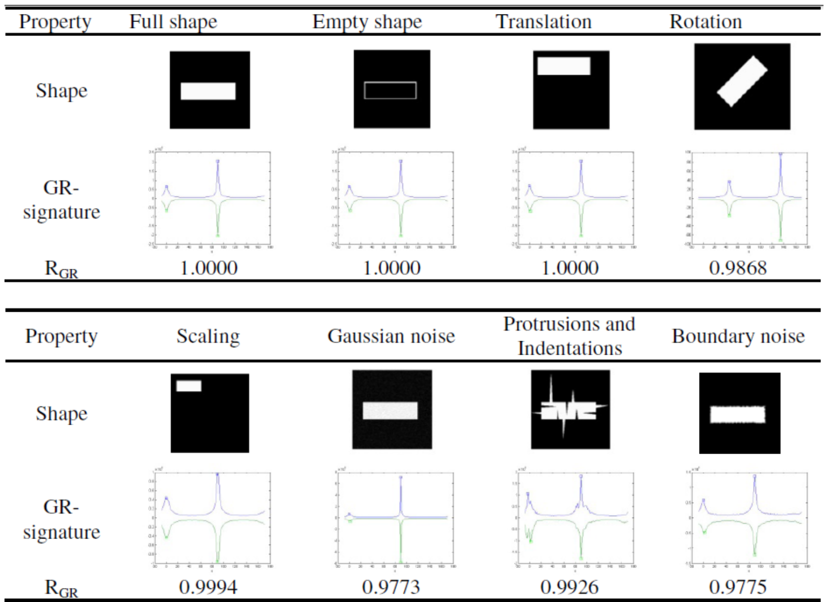


Figure 4: The figure illustrates the properties of GR-signature. Source: [2]

Images' database	Rosin rank	GR-signature rank
Face 1	9	44
Oval shape	12	49
Face 2	13	56
Tree	14	27
Guitar	16	45
Snow crystal	18	30
Maple leaf	21	50
Africa map	26	47
Sword	30	4
Noised rectangle 1	25	9
Noised rectangle 2	55	10
Noised rectangle 3	23	12
Noised rectangle 4	24	13
Noised rectangle 5	40	21
Noised rectangle 6	35	22
Noised rectangle 7	41	24

Figure 5: The figure represents the comparison between results from GR-signature and results from Rosin [5]. The smaller rank the shape has the closer to a rectangle. Source: [2]

4. Conclusion and Perspectives

The paper has presented an approach to measure rectangularity using GR-signature and R_{GR} metric. The experiment has shown, that the R_{GR} metric applied to an ideal rectangle will be evaluated to "1.0" (full shapes, empty shapes,translation). The paper has used Radon transform to recognize shapes and for further generate the GR-Signature. Gradient has been used to locate the peaks and find the modes of density from the output of Radon Transform. Then, we can use GR-signatures together with the rectangularity measurement metric to estimate R_{GR} value. This value can be used to distinguish objects. Finally, the experiment has shown GR-Signature approach has presented better results in comparison to the method presented by Rosin [5].

References

[1] Brady, M. L., Yong, W., 1992. Fast parallel discrete approximation algorithms for the radon transform. In: Proceedings of the fourth annual ACM symposium on Parallel algorithms and architectures. SPAA '92. ACM, New York, NY, USA, pp. 91–99.
URL `http://doi.acm.org/10.1145/140901.140911`

[2] Hentati, J., Naouai, M., Hamouda, A., Weber, C., 2011. Measuring rectangularity using gr-signature. In: MCPR. pp. 136–145.

[3] Lehmann, T. M., Scholl, I., Spitzer, K. (Eds.), 1996. Bildverarbeitung für die Medizin: Algorithmen, Systeme, Anwendungen, Proceedings des Aachener Workshops am 8. und 9. November 1996. Vol. 6 of CEUR Workshop Proceedings. CEUR-WS.org.

[4] Osgood, B. G., Jul. 2008. Lecture 30 - tomography and inverting the radon transform.
URL `http://www.youtube.com/watch?v=xSUHVojA404`

[5] Rosin, P. L., 1999. Measuring rectangularity. Mach. Vis. Appl. 11 (4), 191–196.

[6] Sonka, M., Hlavac, V., Boyle, R., 2007. Image Processing, Analysis, and Machine Vision. Thomson-Engineering.

[7] Sun, J., Xu, Z., Shum, H.-Y., 2008. Image super-resolution using gradient profile prior. In: CVPR.

[8] Tabbone, S., Wendling, L., Salmon, J.-P., Apr. 2006. A new shape descriptor defined on the radon transform. Comput. Vis. Image Underst. 102 (1), 42–51.
URL `http://dx.doi.org/10.1016/j.cviu.2005.06.005`